Further Pure Mathematics FP2 Mark scheme

Question		Sch	eme		Mark		
1		<u>x</u>	<				
	$\frac{x}{x+2} < \frac{2}{x+5}$						
			Seen anywh	ere in solution	B1		
	Critical Values –2 and –5			t B1B1; one correct	B1		
			B1B0				
	$\frac{x}{x+2} - \frac{2}{x+5} < 0$						
	x+2 x+5						
	$x^2 + 3x - 4 < 0$						
	$\frac{x^2 + 3x - 4}{(x+2)(x+5)} < 0$						
	(x+4)(x-1)		Attempt sin	gle fraction and factorise			
	$\frac{(x+4)(x-1)}{(x+2)(x+5)} < 0$		-	or use quad formula	M1		
				-			
	Critical values -4 and 1			ical values May be seen or number line.	A1		
		dM1 · A	0 1	erval inequality using one			
			or -5 with and	1 1 0			
		A1, A1	: Correct inte	rvals	-		
	-5 < x < -4, -2 < x < 1		Can be in set notation		dM1		
	$(-5,-4)\cup(-2,1)$	One con	rrect scores A	1A0	A1		
			Award on basis of the inequalities seen -		A1		
			ore any and/or between them				
			et notation answers do not need the union				
		sign.			(7)		
	Alternative				(7)		
		5	Soon on with	are in colution			
	Critical Values –2 and		-	ere in solution	B1, B		
	$\frac{x}{x+2} < \frac{2}{x+5} \Longrightarrow x(x+5)^2 (x)$	+2) < 2(y)	$(x+2)^2(x+5)$				
	$\Rightarrow (x+5)(x+2)[x(x+5)-2]$	2(x+2)	< 0				
	$\Rightarrow (x+5)(x+2)[(x-1)(x+4)] < 0$		Multi	ply by $(x+5)^{2}(x+2)^{2}$			
			and attempt to factorise a quartic or use quad formula		M1		
	Critical values -4 and 1			et critical values	A1		
				Attempt an interval			
	-5 < x < -4, -2 < x < 1		-	ality using one of -2 or th another cv	dM		
	$(-5,-4)\cup(-2,1)$			1: Correct intervals	A1		
	(-, .) - (-, .)			e in set notation	A1		
				orrect scores A1A0			
				Any solutions with no algebra (eg sketch graph followed by critical values with no working) scores max B1B1			
			ch graph follo	owed by critical values			

Question	Scheme		
2 (a)	$\frac{1}{(r+6)(r+8)}$		
	$\frac{1}{2(r+6)} - \frac{1}{2(r+8)}$ oe Correct partial fractions, any equivalent form		
		(1)	
(b)	$= \left(2 \times \frac{1}{2}\right) \left(\frac{1}{7} - \frac{1}{9} + \frac{1}{8} - \frac{1}{10} + \frac{1}{9} - \frac{1}{11} \dots + \frac{1}{n+5} - \frac{1}{n+7} + \frac{1}{n+6} - \frac{1}{n+8}\right)$ Expands at least 3 terms at start and 2 at end (may be implied) The partial fractions obtained in (a) can be used without multiplying by 2. Fractions may be $\frac{1}{2} \times \frac{1}{7} - \frac{1}{2} \times \frac{1}{9}$ etc These comments apply to both M1 and A1	M1	
	$=\frac{1}{7} + \frac{1}{8} - \frac{1}{n+7} - \frac{1}{n+8}$ Identifies the terms that do not cancel	A1	
	$=\frac{15(n+7)(n+8)-56(2n+15)}{56(n+7)(n+8)}$ Attempt common denominator Must have multiplied the fractions from (a) by 2 now	M1	
	$=\frac{n(15n+113)}{56(n+7)(n+8)}$	A1 cso	
		(4)	
		(5 marks)	

Question	So	cheme	Marks	
3(a)	$\frac{\mathrm{d}y}{\mathrm{d}x} + 2xy = x\mathrm{e}^{-x^2}y^3$			
	$z = y^{-2} \Longrightarrow y = z^{-\frac{1}{2}}$			
	$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{2} z^{-\frac{3}{2}} \frac{\mathrm{d}z}{\mathrm{d}x}$	M1: $\frac{dy}{dx} = kz^{-\frac{3}{2}}\frac{dz}{dx}$	M1 A1	
		A1: Correct differentiation		
	$-\frac{1}{2}z^{-\frac{3}{2}}\frac{\mathrm{d}z}{\mathrm{d}x} + \frac{2x}{z^{\frac{1}{2}}} = x\mathrm{e}^{-x^{2}}z^{-\frac{3}{2}}$	Substitutes for dy/dx	M1	
	$\frac{\mathrm{d}z}{\mathrm{d}x} - 4xz = -2x\mathrm{e}^{-x^2} *$	Correct completion to printed answer with no errors seen	A1 cso	
			(4)	
	Alternative 1		1	
	$\frac{\mathrm{d}z}{\mathrm{d}v} = -2y^{-3}$ oe	$M1: \frac{dz}{dy} = ky^{-3}$	M1 A1	
		A1: Correct differentiation		
	$-\frac{1}{2}y^{3}\frac{dz}{dx} + 2xy = xe^{-x^{2}}y^{3}$	Substitutes for dy/dx	M1	
	$\frac{dz}{dx} - 4xz = -2xe^{-x^2} *$ Correct completion to printed answer with no errors seen		A1	
	Alternative 2		1	
	$\frac{\mathrm{d}z}{\mathrm{d}x} = -2y^{-3}\frac{\mathrm{d}y}{\mathrm{d}x}$	M1: $\frac{dz}{dx} = ky^{-3}\frac{dy}{dx}$ inc chain rule	M1 A1	
		A1: Correct differentiation		
	$-\frac{1}{2}y^{3}\frac{dz}{dx} + 2xy = xe^{-x^{2}}y^{3}$	Substitutes for dy/dx	M1	
	$\frac{\mathrm{d}z}{\mathrm{d}x} - 4xz = -2x\mathrm{e}^{-x^2} *$	Correct completion to printed answer with no errors seen	A1	
(b)	$I = \mathrm{e}^{\int -4x\mathrm{d}x} = \mathrm{e}^{-2x^2}$	M1: $I = e^{\int \pm 4x dx}$	– M1 A1	
-		A1: e^{-2x^2}		
	$z e^{-2x^2} = \int -2x e^{-3x^2} dx$	$z \times I = \int -2x \mathrm{e}^{-x^2} I \mathrm{d}x$	dM1	
	$\frac{1}{3}e^{-3x^2}(+c)$	$\int x \mathrm{e}^{q x^2} \mathrm{d}x = p \mathrm{e}^{q x^2} \left(+c\right)$	M1	
	$z = ce^{2x^2} + \frac{1}{3}e^{-x^2}$	Or equivalent	A1	
			(5)	

269

Question	Scheme	
3(c)	$\frac{1}{y^2} = ce^{2x^2} + \frac{1}{3}e^{-x^2} \Longrightarrow y^2 = \frac{1}{ce^{2x^2} + \frac{1}{3}e^{-x^2}} \qquad y^2 = \frac{1}{(b)} \left(= \frac{3e^{x^2}}{1 + ke^{3x^2}} \right)$	B1ft
		(1)
	(1)	0 marks)

Question	Scheme		Marks
4(a)	$w = \frac{z - 1}{z + 1}$		
	$w = \frac{z - 1}{z + 1} \Longrightarrow wz + w = z - 1 \Longrightarrow z = \dots$	Attempt to make z the subject	M1
	$z = \frac{w+1}{1-w}$	Correct expression in terms of w	A1
	$=\frac{u+iv+1}{1-u-iv}\times\frac{1-u+iv}{1-u+iv}$	Introduces " $u + iv$ " and multiplies top and bottom by the complex conjugate of the bottom	M1
	$x = \frac{-u^2 - v^2 + 1}{\dots}, y = \frac{2v}{\dots}$		
	$y = 2x \Longrightarrow 2v = -2u^2 - 2v^2 + 2$	Uses real and imaginary parts and y = 2x to obtain an equation connecting "u" and "v" Can have the 2 on the wrong side.	M1
	$u^2 + \left(v + \frac{1}{2}\right)^2 - \frac{1}{4} = 1$	Processes their equation to a form that is recognisable as a circle ie coefficients of u^2 and v^2 are the same and no uv terms	M1
	Centre $(0, -\frac{1}{2})$, radius $\frac{\sqrt{5}}{2}$	A1: Correct centre (allow - ¹ / ₂ i) A1: Correct radius	A1,A1
			(7)
	Special Case: $w = \frac{x + iy - 1}{x + iy + 1} = \frac{(x - 1) + 2xi}{(x + 1) + 2xi} \times \frac{(x + 1) - 2xi}{(x + 1) - 2xi}$	M1: rationalise the denominator, may have $2x$ or y	
	$=\frac{(x^{2}-1)+4x^{2}+2xi(x+1-(x-1))}{(x+1)^{2}+4x^{2}}$	A1: Correct result in terms of <i>x</i> only. Must have rational denominator shown, but no other simplification needed	
(b)		B1ft: Their circle correctly positioned provided their equation does give a circle	B1ft B1
	R	B1: Completely correct sketch and shading	DINDI
		1	(2)
		(9 marks)

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271

5(a) $y = \cot x$ $\frac{dy}{dx} = -\csc^{2}x$ $\frac{dy}{dx^{2}} = (-2\csc x)(-\csc x \cot x)$ $\frac{d^{2}y}{dx^{2}} = (-2\csc x)(-\csc x \cot x)$ $M1: Differentiates using the chain M1: Alt Correct derivative M1: Correct completion to printed answer M1: Verture M1: Verture derivative m2: Verture derivative m3: Verture derivative derivat$	Question	Scheme		
$\frac{d^2 y}{dx^2} = (-2\csc x)(-\csc x \cot x)$ $\frac{M1: Differentiates using the chain rule or product/quotient rule All: Correct derivative All: Correct completion to printed answer 1 + cot2 x = cosec2 x or cos2 x = 2 cot x + 2 cot3 x* = 2cosec2 x cot x = 2 cot x + 2 cot3 x* \frac{A1: Correct completion to printed answer x = 1 + cot2 x = cosec2 x or cos2 x + sin2 x = 1 must be used Full working must be shown (3) \frac{d^2 y}{dx^2} = -(-2\sin^{-3} x \cos x) = \frac{d^2 y}{dx^2} = -(-2\sin^{-3} x \cos x) = \frac{M1A1}{(3)} \frac{d^3 y}{dx^2} = -2\csc^2 x - 6\cot^2 x \csc^2 x \frac{Correct third derivative}{(3)} \frac{B1}{dx^2} = -2\csc^2 x - 6\cot^2 x (1 + cot^2 x) \frac{Uses 1 + cot^2 x = cosec^2 x}{(3)} \frac{(6)}{f(\frac{\pi}{3}) = \frac{1}{\sqrt{3}}, f'(\frac{\pi}{3}) = -\frac{4}{3}, f''(\frac{\pi}{3}) = \frac{8}{3\sqrt{3}}, f'''(\frac{\pi}{3}) = -\frac{16}{3} \frac{(7)}{M1: Attempts all 4 values at \frac{\pi}{3} No working need be shown \frac{(y =)\frac{1}{\sqrt{3}} - \frac{4}{3}(x - \frac{\pi}{3}) + \frac{4}{3\sqrt{3}}(x - \frac{\pi}{3})^2 - \frac{8}{9}(x - \frac{\pi}{3})^3 M1: Correct application of Taylor using their values. Must be up to and including \left(x - \frac{\pi}{3}\right)^3 M1A1$	5(a)	$y = \cot x$		
$\frac{d^{2}y}{dx^{2}} = (-2\csc x)(-\csc x \cot x)$ $\frac{\operatorname{rule or product/quotient rule}}{A1: \operatorname{Correct derivative}} $ $\frac{d^{2}y}{dx^{2}} = (-2\csc x \cot x = 2\cot x + 2\cot^{3} x^{*})$ $\frac{A1: \operatorname{Correct completion to printed answer}}{1 + \cot^{2} x = \csc^{2} x \text{ or } \cos^{2} x + \sin^{2} x = 1} $ $\operatorname{must be used}$ $\frac{d^{2}y}{\sin x} \rightarrow \frac{dy}{dx} = \frac{-\sin^{2} x - \cos^{2} x}{\sin^{2} x} = -\frac{1}{\sin^{2} x}$ $\frac{d^{2}y}{dx^{2}} = -(-2\sin^{-3} x \cos x) = \dots$ $\frac{d^{2}y}{dx^{2}} = -(2\sin^{-3} x \cos x) = \dots$ $\frac{d^{2}y}{dx^{2}} = -2\csc^{2} x - 6\cot^{2} x \csc^{2} x$ $\frac{Correct third derivative}{(3)}$ $\frac{d^{2}y}{dx^{3}} = -2\csc^{2} x - 6\cot^{2} x(1 + \cot^{2} x)$ $\frac{Uses 1 + \cot^{2} x = \csc^{2} x}{(3)}$ $\frac{d^{2}y}{dx^{2}} = -2\cot^{4} x - 8\cot^{2} x - 2$ $\frac{d^{2}y}{dx^{3}} = -\frac{1}{\sqrt{3}}, f'(\frac{\pi}{3}) = -\frac{4}{3}, f''(\frac{\pi}{3}) = \frac{8}{3\sqrt{3}}, f'''(\frac{\pi}{3}) = -\frac{16}{3}$ $\frac{d^{2}y}{dx^{2}} = -\frac{4}{3} (x - \frac{\pi}{3})^{2} - \frac{8}{9} (x - \frac{\pi}{3})^{3}$ $\frac{d^{2}y}{dx^{2}} = -\frac{4}{3} (x - \frac{\pi}{3})^{2} - \frac{8}{9} (x - \frac{\pi}{3})^{3}$ $\frac{d^{2}y}{dx^{2}} = -\frac{4}{3} (x - \frac{\pi}{3})^{2} - \frac{8}{9} (x - \frac{\pi}{3})^{3}$ $\frac{d^{2}y}{dx^{2}} = -\frac{4}{3} (x - \frac{\pi}{3})^{2} - \frac{8}{9} (x - \frac{\pi}{3})^{3}$ $\frac{d^{2}y}{dx^{2}} = -\frac{4}{3} (x - \frac{\pi}{3})^{2} - \frac{8}{9} (x - \frac{\pi}{3})^{3}$ $\frac{d^{2}y}{dx^{2}} = -\frac{4}{3} (x - \frac{\pi}{3})^{2} - \frac{8}{9} (x - \frac{\pi}{3})^{3}$ $\frac{d^{2}y}{dx^{2}} = -\frac{4}{3} (x - \frac{\pi}{3})^{2} - \frac{8}{9} (x - \frac{\pi}{3})^{3}$ $\frac{d^{2}y}{dx^{2}} = -\frac{4}{3} (x - \frac{\pi}{3})^{2} - \frac{8}{9} (x - \frac{\pi}{3})^{3}$ $\frac{d^{2}y}{dx^{2}} = -\frac{4}{3} (x - \frac{\pi}{3})^{2} - \frac{8}{9} (x - \frac{\pi}{3})^{3}$ $\frac{d^{2}y}{dx^{2}} = -\frac{1}{2} + \frac{1}{2} + $		$\frac{\mathrm{d}y}{\mathrm{d}x} = -\mathrm{cosec}^2 x$		
$\begin{array}{ c c c c c c } \hline & A1: Correct completion to printed answer \\ & + \cot^2 x = \operatorname{cosec}^2 x \text{ or } x = 2 \cot x + 2 \cot^3 x^* & \begin{array}{c} A1: \operatorname{correct} x + \operatorname{cosec}^2 x \text{ or } \\ \cos^2 x + \sin^2 x = 1 \\ \operatorname{must} be used \\ \hline & \operatorname{Full working must} be shown \end{array} \end{array} $ $\begin{array}{ c c c c c c c c } \hline & & & & & & & & & & & & & & & & & & $		$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = (-2\operatorname{cosec} x)(-\operatorname{cosec} x \cot x)$	rule or product/quotient rule	M1A1
Alternative $y = \frac{\cos x}{\sin x} \rightarrow \frac{dy}{dx} = \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = -\frac{1}{\sin^2 x}$ $\frac{d^2 y}{dx^2} = -(-2\sin^{-3}x\cos x) =$ M1A1Correct completion to printed answer see aboveA1(a)(b) $\frac{d^3 y}{dx^3} = -2\csc^2 x - 6\cot^2 x \csc^2 x$ Correct third derivativeB1 $= -2(1+\cot^2 x) - 6\cot^2 x(1+\cot^2 x)$ Uses $1+\cot^2 x = \csc^2 x$ M1 $= -6\cot^4 x - 8\cot^2 x - 2$ (c) $f(\frac{\pi}{3}) = \frac{1}{\sqrt{3}}, f'(\frac{\pi}{3}) = -\frac{4}{3}, f''(\frac{\pi}{3}) = \frac{8}{3\sqrt{3}}, f'''(\frac{\pi}{3}) = -\frac{16}{3}$ M1M1: Attempts all 4 values at $\frac{\pi}{3}$ No working need be shown $(y =) \frac{1}{\sqrt{3}} - \frac{4}{3}(x - \frac{\pi}{3}) + \frac{4}{3\sqrt{3}}(x - \frac{\pi}{3})^2 - \frac{8}{9}(x - \frac{\pi}{3})^3$ M1: Correct application of Taylor using their values. Must be up to and including $\left(x - \frac{\pi}{3}\right)^3$ M1: A1: Correct expression Must start $y =$ or $\cot x$ f(x) allowed provided defined here or above as $f(x) = \cot x$ or yDecimal equivalents allowed (min 3 sf apart from 0.77), 0.578, 1.33, 0.770, (0.7698, so accept 0.77) 0.889		$= 2\operatorname{cosec}^2 x \cot x = 2 \cot x + 2 \cot^3 x^*$	A1: Correct completion to printed answer $1 + \cot^2 x = \csc^2 x$ or $\cos^2 x + \sin^2 x = 1$ must be used	A1cso*
$y = \frac{\cos x}{\sin x} \rightarrow \frac{dy}{dx} = \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = -\frac{1}{\sin^2 x}$ $\frac{d^2 y}{dx^2} = -(-2\sin^{-3}x\cos x) = \dots$ M1A1 Correct completion to printed answer see above A1 (3) (b) $\frac{d^3 y}{dx^3} = -2\csc^2 x - 6\cot^2 x \csc^2 x$ Correct third derivative B1 $= -2(1 + \cot^2 x) - 6\cot^2 x(1 + \cot^2 x)$ Uses $1 + \cot^2 x = \csc^2 x$ M1 $= -6\cot^4 x - 8\cot^2 x - 2$ (3) (c) $f\left(\frac{\pi}{3}\right) = \frac{1}{\sqrt{3}}, f'\left(\frac{\pi}{3}\right) = -\frac{4}{3}, f''\left(\frac{\pi}{3}\right) = \frac{8}{3\sqrt{3}}, f'''\left(\frac{\pi}{3}\right) = -\frac{16}{3}$ M1 M1: Attempts all 4 values at $\frac{\pi}{3}$ No working need be shown ($y =)\frac{1}{\sqrt{3}} - \frac{4}{3}\left(x - \frac{\pi}{3}\right)^2 + \frac{4}{3\sqrt{3}}\left(x - \frac{\pi}{3}\right)^2 - \frac{8}{9}\left(x - \frac{\pi}{3}\right)^3$ M1: Correct application of Taylor using their values. Must be up to and including $\left(x - \frac{\pi}{3}\right)^3$ M1A1 A1: Correct expression Must start $y = \dots$ or $\cot x$ f(x) allowed provided defined here or above as $f(x) = \cot x$ or y Decimal equivalents allowed (min 3 sf apart from 0.77), 0.578, 1.33, 0.770, (0.7698, so accept 0.77) 0.889				(3)
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$\frac{d^{3}}{dx^{3}} = -2 \operatorname{cosec}^{2} x - 6 \operatorname{cot}^{2} x \operatorname{cosec}^{2} x$ $\frac{d^{3} y}{dx^{3}} = -2 \operatorname{cosec}^{2} x - 6 \operatorname{cot}^{2} x \operatorname{cosec}^{2} x$ $\frac{d^{3} y}{dx^{3}} = -2 \operatorname{cosec}^{2} x - 6 \operatorname{cot}^{2} x \operatorname{cosec}^{2} x$ $\frac{d^{3} y}{dx^{3}} = -2 \operatorname{cosec}^{2} x - 6 \operatorname{cot}^{2} x \operatorname{cosec}^{2} x$ $\frac{d^{3} y}{dx^{3}} = -2 \operatorname{cosec}^{2} x - 6 \operatorname{cot}^{2} x \operatorname{cosec}^{2} x$ $\frac{d^{3} y}{dx^{3}} = -2 \operatorname{cosec}^{2} x - 6 \operatorname{cot}^{2} x (1 + \operatorname{cot}^{2} x))$ $\frac{d^{3} y}{dx^{3}} = -2 \operatorname{cosec}^{2} x - 6 \operatorname{cot}^{2} x (1 + \operatorname{cot}^{2} x))$ $\frac{d^{3} y}{dx^{3}} = -2 \operatorname{cosec}^{2} x - 6 \operatorname{cot}^{2} x (1 + \operatorname{cot}^{2} x))$ $\frac{d^{3} y}{dx^{3}} = -2 \operatorname{cosec}^{2} x - 6 \operatorname{cot}^{2} x (1 + \operatorname{cot}^{2} x))$ $\frac{d^{3} y}{dx^{3}} = -2 \operatorname{cosec}^{2} x - 6 \operatorname{cot}^{2} x (1 + \operatorname{cot}^{2} x))$ $\frac{d^{3} y}{dx^{3}} = -2 \operatorname{cosec}^{2} x - 2 \operatorname{cosec}^{2} x$ $\frac{d^{3} y}{dx^{3}} = -6 \operatorname{cot}^{4} x - 8 \operatorname{cot}^{2} x - 2$ $\frac{d^{3} y}{dx^{3}} = -\frac{1}{\sqrt{3}} \operatorname{cosec}^{2} x - 2$ $\frac{d^{3} y}{dx^{3}} = -\frac{1}{\sqrt{3}} co$		$y = \frac{\cos x}{\sin x} \rightarrow \frac{dy}{dx} = \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = -$	$\frac{1}{\sin^2 x}$	
(b) $\frac{d^{3}y}{dx^{3}} = -2\csc^{2}x - 6\cot^{2}x\csc^{2}x$ Correct third derivative B1 $= -2(1 + \cot^{2}x) - 6\cot^{2}x(1 + \cot^{2}x)$ Uses $1 + \cot^{2}x = \csc^{2}x$ M1 $= -6\cot^{4}x - 8\cot^{2}x - 2$ cso A1 (3) (c) $f\left(\frac{\pi}{3}\right) = \frac{1}{\sqrt{3}}, f'\left(\frac{\pi}{3}\right) = -\frac{4}{3}, f''\left(\frac{\pi}{3}\right) = \frac{8}{3\sqrt{3}}, f'''\left(\frac{\pi}{3}\right) = -\frac{16}{3}$ M1 M1: Attempts all 4 values at $\frac{\pi}{3}$ No working need be shown $(y =)\frac{1}{\sqrt{3}} - \frac{4}{3}\left(x - \frac{\pi}{3}\right) + \frac{4}{3\sqrt{3}}\left(x - \frac{\pi}{3}\right)^{2} - \frac{8}{9}\left(x - \frac{\pi}{3}\right)^{3}$ M1: Correct application of Taylor using their values. Must be up to and including $\left(x - \frac{\pi}{3}\right)^{3}$ M1A1 A1: Correct expression Must start $y = \dots$ or cot x f(x) allowed provided defined here or above as $f(x) = \cot x$ or y Decimal equivalents allowed (min 3 sf apart from 0.77), 0.578, 1.33, 0.770, (0.7698, so accept 0.77) 0.889		$\frac{d^2 y}{dx^2} = -(-2\sin^{-3} x \cos x) = \dots$		M1A1
(b) $\frac{d^3y}{dx^3} = -2\csc^2 x - 6\cot^2 x \csc^2 x$ Correct third derivative B1 $= -2(1 + \cot^2 x) - 6\cot^2 x(1 + \cot^2 x)$ Uses $1 + \cot^2 x = \csc^2 x$ M1 $= -6\cot^4 x - 8\cot^2 x - 2$ cso A1 (3) (c) $f\left(\frac{\pi}{3}\right) = \frac{1}{\sqrt{3}}, f'\left(\frac{\pi}{3}\right) = -\frac{4}{3}, f''\left(\frac{\pi}{3}\right) = \frac{8}{3\sqrt{3}}, f'''\left(\frac{\pi}{3}\right) = -\frac{16}{3}$ M1 M1: Attempts all 4 values at $\frac{\pi}{3}$ No working need be shown ($y =)\frac{1}{\sqrt{3}} - \frac{4}{3}\left(x - \frac{\pi}{3}\right) + \frac{4}{3\sqrt{3}}\left(x - \frac{\pi}{3}\right)^2 - \frac{8}{9}\left(x - \frac{\pi}{3}\right)^3$ M1: Correct application of Taylor using their values. Must be up to and including $\left(x - \frac{\pi}{3}\right)^3$ M1: Correct expression Must start $y = \dots$ or cot x f(x) allowed provided defined here or above as $f(x) = \cot x$ or y Decimal equivalents allowed (min 3 sf apart from 0.77), 0.578, 1.33, 0.770, (0.7698, so accept 0.77) 0.889		Correct completion to printed answer see above		A1
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$=-6 \cot^{4} x - 8 \cot^{2} x - 2$ $=-6 \cot^{4} x - 8 \cot^{2} x - 2$ (3) $f\left(\frac{\pi}{3}\right) = \frac{1}{\sqrt{3}}, f'\left(\frac{\pi}{3}\right) = -\frac{4}{3}, f''\left(\frac{\pi}{3}\right) = \frac{8}{3\sqrt{3}}, f'''\left(\frac{\pi}{3}\right) = -\frac{16}{3}$ $M1$ $M1: \text{ Attempts all 4 values at } \frac{\pi}{3} \text{ No working need be shown}$ $(y =) \frac{1}{\sqrt{3}} - \frac{4}{3}\left(x - \frac{\pi}{3}\right) + \frac{4}{3\sqrt{3}}\left(x - \frac{\pi}{3}\right)^{2} - \frac{8}{9}\left(x - \frac{\pi}{3}\right)^{3}$ $M1: \text{ Correct application of Taylor using their values. Must be up to and including } \left(x - \frac{\pi}{3}\right)^{3}$ $M1A1$ $A1: \text{ Correct expression Must start } y = \dots \text{ or } \cot x$ $f(x) \text{ allowed provided defined here or above as } f(x) = \cot x \text{ or } y$ $Decimal equivalents allowed (min 3 sf apart from 0.77), 0.578, 1.33, 0.770, (0.7698, so accept 0.77) 0.889$	(b)	$\frac{d^3 y}{dx^3} = -2\csc^2 x - 6\cot^2 x \csc^2 x$	Correct third derivative	B1
(c) $f\left(\frac{\pi}{3}\right) = \frac{1}{\sqrt{3}}, f'\left(\frac{\pi}{3}\right) = -\frac{4}{3}, f''\left(\frac{\pi}{3}\right) = \frac{8}{3\sqrt{3}}, f'''\left(\frac{\pi}{3}\right) = -\frac{16}{3}$ M1 M1: Attempts all 4 values at $\frac{\pi}{3}$ No working need be shown $\left(y=\right)\frac{1}{\sqrt{3}}-\frac{4}{3}\left(x-\frac{\pi}{3}\right)+\frac{4}{3\sqrt{3}}\left(x-\frac{\pi}{3}\right)^2-\frac{8}{9}\left(x-\frac{\pi}{3}\right)^3$ M1: Correct application of Taylor using their values. Must be up to and including $\left(x-\frac{\pi}{3}\right)^3$ A1: Correct expression Must start $y = \dots$ or $\cot x$ $f(x)$ allowed provided defined here or above as $f(x) = \cot x$ or y Decimal equivalents allowed (min 3 sf apart from 0.77), 0.578, 1.33, 0.770, (0.7698, so accept 0.77) 0.889		$= -2(1 + \cot^2 x) - 6\cot^2 x(1 + \cot^2 x)$	Uses $1 + \cot^2 x = \csc^2 x$	M1
(c) $f\left(\frac{\pi}{3}\right) = \frac{1}{\sqrt{3}}, f'\left(\frac{\pi}{3}\right) = -\frac{4}{3}, f''\left(\frac{\pi}{3}\right) = \frac{8}{3\sqrt{3}}, f'''\left(\frac{\pi}{3}\right) = -\frac{16}{3}$ M1: Attempts all 4 values at $\frac{\pi}{3}$ No working need be shown $(y=)\frac{1}{\sqrt{3}} - \frac{4}{3}\left(x - \frac{\pi}{3}\right) + \frac{4}{3\sqrt{3}}\left(x - \frac{\pi}{3}\right)^2 - \frac{8}{9}\left(x - \frac{\pi}{3}\right)^3$ M1: Correct application of Taylor using their values. Must be up to and including $\left(x - \frac{\pi}{3}\right)^3$ M1: Correct expression Must start $y = \dots$ or $\cot x$ $f(x)$ allowed provided defined here or above as $f(x) = \cot x$ or y Decimal equivalents allowed (min 3 sf apart from 0.77), 0.578, 1.33, 0.770, (0.7698, so accept 0.77) 0.889		$=-6\cot^4 x - 8\cot^2 x - 2$	cso	A1
M1 M1: Attempts all 4 values at $\frac{\pi}{3}$ No working need be shown $\left(y=\right)\frac{1}{\sqrt{3}}-\frac{4}{3}\left(x-\frac{\pi}{3}\right)+\frac{4}{3\sqrt{3}}\left(x-\frac{\pi}{3}\right)^2-\frac{8}{9}\left(x-\frac{\pi}{3}\right)^3$ M1: Correct application of Taylor using their values. Must be up to and including $\left(x-\frac{\pi}{3}\right)^3$ A1: Correct expression Must start $y = \dots$ or cot x f(x) allowed provided defined here or above as $f(x) = \cot x$ or y Decimal equivalents allowed (min 3 sf apart from 0.77), 0.578, 1.33, 0.770, (0.7698, so accept 0.77) 0.889				(3)
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(0.7698, so accept 0.77) 0.889		M1: Correct application of Taylor using their values. Must be up to and including $\left(x - \frac{\pi}{3}\right)^3$ A1: Correct expression Must start $y = \dots$ or cot x		M1A1
(3)			apart from 0.77), 0.578, 1.33, 0.770,	
				(3)

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Question	Scheme		
6(a)	$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} - 2\frac{\mathrm{d}y}{\mathrm{d}x} - 3y = 2\sin x$		
	AE: $m^2 - 2m - 3 = 0$		
	$m^2 - 2m - 3 = 0 \Longrightarrow m = \dots (-1, 3)$	Forms Auxiliary Equation and attempts to solve (usual rules)	M1
	$(y=)Ae^{3x}+Be^{-x}$	Cao	A1
	$PI: (y=) p \sin x + q \cos x$	Correct form for PI	B1
	$(y'=) p \cos x - q \sin x$ $(y''=) - p \sin x - q \cos x$		
	$-p\sin x - q\cos x - 2(p\cos x - q\sin x)$ Differentiates twice and substitutes	$-3p\sin x - 3q\cos x = 2\sin x$	M1
	2q-4p=2, 4q+2p=0 Correct equations		A1
	$p = -\frac{2}{5}, q = \frac{1}{5}$	A1A1 both correct A1A0 one correct	A1 A1
	$y = \frac{1}{5}\cos x - \frac{2}{5}\sin x$		
	$y = Ae^{3x} + Be^{-x} + \frac{1}{5}\cos x - \frac{2}{5}\sin x$	Follow through their <i>p</i> and <i>q</i> and their CF	B1ft
			(8)
(b)	$y' = 3Ae^{3x} - Be^{-x} - \frac{1}{5}\sin x - \frac{2}{5}\cos x$	Differentiates their GS	M1
	$0 = A + B + \frac{1}{5}, \ 1 = 3A - B - \frac{2}{5}$	M1: Uses the given conditions to give two equations in A and B	M1 A1
	5 5	A1: Correct equations	
	$A = \frac{3}{10}, B = -\frac{1}{2}$	Solves for <i>A</i> and <i>B</i> Both correct	A1
	$y = \frac{3}{10}e^{3x} - \frac{1}{2}e^{-x} + \frac{1}{5}\cos x - \frac{2}{5}\sin x$	Sub their values of A and B in their GS	A1ft
			(5)
		(1	l3 marks)

Question	Scheme		
7(a)	$\theta = \frac{\pi}{3} \Longrightarrow r = \sqrt{3}\sin\left(\frac{\pi}{3}\right) = \frac{3}{2}$	Attempt to verify coordinates in at least one of the polar equations	M1
	$\theta = \frac{\pi}{3} \Longrightarrow r = 1 + \cos\left(\frac{\pi}{3}\right) = \frac{3}{2}$	Coordinates verified in both curves (Coordinate brackets not needed)	A1
			(2)
	Alternative		
	Equate rs: $\sqrt{3}\sin\theta = 1 + \cos\theta$ and verify (by su solution or solve by using $t = \tan\frac{\theta}{2}$ or writing	, ,	M1
	$\frac{\sqrt{3}}{2}\sin\theta - \frac{1}{2}\cos\theta = \frac{1}{2} \qquad \sin\left(\theta - \frac{\pi}{6}\right) = \frac{1}{2} \theta =$ Squaring the original equation allowed as θ is π	5	
	Use $\theta = \frac{\pi}{3}$ in either equation to obtain $r = \frac{3}{2}$		A1
(b)	$\frac{1}{2}\int (\sqrt{3}\sin\theta)^2 \mathrm{d}\theta, \frac{1}{2}\int (1+\cos\theta)^2 \mathrm{d}\theta$	Correct formula used on at least one curve (1/2 may appear later) Integrals may be separate or added or subtracted.	(2) M1
	$=\frac{1}{2}\int 3\sin^2\theta \mathrm{d}\theta, \frac{1}{2}\int (1+2\cos\theta+\cos^2\theta)\mathrm{d}\theta$		
	$= \left(\frac{1}{2}\right) \int \frac{3}{2} (1 - \cos 2\theta) d\theta, \left(\frac{1}{2}\right) \int (1 + 2\cos \theta + \frac{1}{2}(1 + \cos 2\theta)) d\theta$ Attempt to use $\sin^2 \theta$ or $\cos^2 \theta = \pm \frac{1}{2} \pm \frac{1}{2} \cos 2\theta$ on either integral Not dependent 1/2 may be missing		
	$=\frac{3}{4}\left[\theta - \frac{1}{2}\sin 2\theta\right]_{(0)}^{\left(\frac{\pi}{3}\right)}, \frac{1}{2}\left[\frac{3}{2}\theta + 2\sin \theta + \frac{1}{4}\sin 2\theta\right]_{\left(\frac{\pi}{3}\right)}^{\left(\pi\right)}$		
	Correct integration (ignore limits) A1A1 or A $R = \frac{3}{4} \left[\frac{\pi}{3} - \frac{\sqrt{3}}{4} \left(-0 \right) \right] + \frac{1}{2} \left[\frac{3\pi}{2} - \left(\frac{\pi}{2} + \sqrt{3} + \frac{\sqrt{3}}{8} \right) \right]$	Correct use of limits for both	dd M1
	$=\frac{3}{4}(\pi-\sqrt{3})$	Cao No equivalents allowed	A1
	<u> </u>		(6)
	1	1	(8 marks)

PMT

Question	Scheme			Marks
8(a)	$\left(z+\frac{1}{z}\right)^3 \left(z-\frac{1}{z}\right)^3 = \left(z^2-\frac{1}{z^2}\right)^3$			
	$=z^6 - 3z^2 + \frac{3}{z^2} - z^{-6}$		1: Attempt to expand : Correct expansion	M1A1
	$=z^{6} - \frac{1}{z^{6}} - 3\left(z^{2} - \frac{1}{z^{2}}\right)$	Co see	rrect answer with no errors	A1
	/			(3)
	Alternative			
	$\left(z+\frac{1}{z}\right)^{3} = z^{3}+3z+\frac{3}{z}+\frac{1}{z^{3}}, \left(z-\frac{1}{z}\right)^{3} = z^{3}$	-32	$z + \frac{3}{z} - \frac{1}{z^3}$	M1A1
	M1: Attempt to expand both cubic brackets A	.1: (Correct expansions	
	$= z^{6} - \frac{1}{z^{6}} - 3\left(z^{2} - \frac{1}{z^{2}}\right)$		Correct answer with no errors	A1
				(3)
(b)(i)(ii)	$z^n = \cos n\theta + i\sin n\theta$		Correct application of de Moivre	B1
	$z^{-n} = \cos(-n\theta) + i\sin(-n\theta) = \pm \cos n\theta \pm \sin n$ but must be different from their z^n	$z^{-n} = \cos(-n\theta) + i\sin(-n\theta) = \pm \cos n\theta \pm \sin n\theta$ but must be different from their z^n Attempt z^{-n}		M1
	$z^{n} + \frac{1}{z^{n}} = 2\cos n\theta^{*}, \ z^{n} - \frac{1}{z^{n}} = 2i\sin n\theta^{*} \qquad z^{-n} = \cos n\theta - i\sin n\theta$ must be seen		A1*	
				(3)
(c)	$\left(z+\frac{1}{z}\right)^{3}\left(z-\frac{1}{z}\right)^{3} = \left(2\cos\theta\right)^{3}\left(2i\sin\theta\right)^{3}$			B1
	$z^{6} - \frac{1}{z^{6}} - 3\left(z^{2} - \frac{1}{z^{2}}\right) = 2i\sin 6\theta - 6i\sin 2\theta$		ollow through their <i>k</i> in lace of 3	B1ft
	$-64i\sin^3\theta\cos^3\theta = 2i\sin6\theta - 6i\sin2\theta$	si	quating right hand sides and mplifying $2^3 \times (2i)^3$ (B mark eeded for each side to gain	M1
	M mark)			
	$\cos^3\theta\sin^3\theta = \frac{1}{32}(3\sin 2\theta - \sin 6\theta) *$			Alcso
				(4)

Question	Scheme		Marks
8(d)	$\int_{0}^{\frac{\pi}{8}} \cos^{3}\theta \sin^{3}\theta \mathrm{d}\theta = \int_{0}^{\frac{\pi}{8}} \frac{1}{32} (3\sin 2\theta - \sin 6\theta) \mathrm{d}\theta$		
	π	M1: $p\cos 2\theta + q\cos 6\theta$	
	$=\frac{1}{32}\left[-\frac{3}{2}\cos 2\theta + \frac{1}{6}\cos 6\theta\right]_{0}^{\frac{1}{8}}$	A1: Correct integration Differentiation scores M0A0	M1 A1
	$=\frac{1}{32}\left[\left(-\frac{3}{2\sqrt{2}}-\frac{1}{6\sqrt{2}}\right)-\left(-\frac{3}{2}+\frac{1}{6}\right)\right]=\frac{1}{32}\left(\frac{4}{3}-\frac{5\sqrt{2}}{6}\right)$	dM1: Correct use of limits – lower limit to have non- zero result. Dep on previous M mark	dM1 A1
		A1: Cao (oe) but must be exact	
			(4)
		(1	4 marks)